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N=2 Curves and a Coulomb Phase in N=1 SUSY Gauge Theories with Adjoint and Fundamental Matters

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Abstract

We study low-energy effective superpotentials for the phase with a confined photon in $N = 1$ supersymmetric gauge theories with an adjoint matter Φ and fundamental flavors $Q, \tilde{Q}$. Arbitrary classical gauge groups are considered. The results are used to derive the hyperelliptic curves which describe the Coulomb phase of $N = 2$ supersymmetric QCD with classical gauge groups. These curves are in agreement with those proposed earlier by several authors. Our results also produce the curves relevant to describe the Coulomb phase of $N = 1$ theories with a superpotential of the form $\tilde{Q}\Phi^l Q$.

In the last three years, there has been a significant development in understanding four-dimensional supersymmetric gauge theories [1],[2],[3]. In particular new light is shed on mechanism for confinement in view of condensation of massless solitons in the vacua of the Coulomb phase in $N = 2$ supersymmetric gauge theories. It is important that these massless solitons appear at the singularities of moduli space of vacua and the superpotential which breaks $N = 2$ supersymmetry to $N = 1$ causes condensation. Conversely, from the analysis of the confining phase of $N = 1$ theories with a suitable superpotential, we can identify the singular points in the Coulomb phase of $N = 2$ theories. Thus it becomes possible to construct the $N = 2$ Seiberg-Witten curves since the $N = 2$ curves are determined almost completely by the singularity conditions. This idea has been successfully applied to $N = 2$ supersymmetric Yang-Mills theories with the classical groups as well as G_2, and $N = 2$ supersymmetric $SU(N_c)$ QCD [4]-[11].

In this paper, we extend our analysis to describe the Coulomb phase of $N = 1$ supersymmetric gauge theories with an adjoint matter field Φ and fundamental flavors $Q, \tilde{Q}$. A tree-level superpotential consists of the Yukawa-like term $\tilde{Q}\Phi^l Q$ in addition to the Casimir terms built out of Φ, and we shall consider arbitrary classical gauge groups. In the appropriate limit the theory is reduced to $N = 2$ supersymmetric QCD. We derive the low-energy effective superpotential for the phase with a single confined photon and obtain the hyperelliptic curves which describe the Coulomb phase of the theory. In the $N = 2$ limit we will show that these curves agree with the results of [12]-[15], and hence our results are also viewed as a non-trivial test of the $N = 2$ curves proposed previously.

We start with discussing $N = 1$ $SU(N_c)$ supersymmetric gauge theory with an adjoint matter field Φ, N_f flavors of fundamentals Q and anti-fundamentals $\tilde{Q}$ to explain our method in this paper. We take a tree-level superpotential

$$W = \sum_{n=1}^{N_c} g_n u_n + \sum_{l=0}^{r} \mathrm{Tr}_{N_f}\, \lambda_l\, \tilde{Q}\Phi^l Q, \qquad u_n = \frac{1}{n}\mathrm{Tr}\, \Phi^n, \tag{1}$$

where $\mathrm{Tr}_{N_f}\, \lambda_l\, \tilde{Q}\Phi^l Q = \sum_{i,j=1}^{N_f} (\lambda_l)^i_j \tilde{Q}_i \Phi^l Q^j$ and $r \leq N_c - 1$. If we set $(\lambda_0)^i_j = m^i_j$ with $[m, m^\dagger] = 0$, $(\lambda_1)^i_j = \delta^i_j$, $(\lambda_l)^i_j = 0$ for $l > 1$ and all $g_i = 0$, eq.(1) recovers the superpotential in $N = 2$ $SU(N_c)$ supersymmetric QCD with quark mass m. The second term in (1) was considered in a recent work [16].

Let us focus on the classical vacua with $Q = \tilde{Q} = 0$ and an unbroken $SU(2) \times U(1)^{N_c-2}$ symmetry which means $\Phi = \mathrm{diag}(a_1, a_1, a_2, a_3, \cdots, a_{N_c-1})$ up to gauge transformations. (Note that the superpotential (1) has no classical vacua with unbroken $U(1)^{N_c-1}$.) In this vacuum, we will evaluate semiclassicaly the low-energy effective superpotential. Our procedure is slightly different from that adopted in [8] upon treating Q and $\tilde{Q}$. We investigate the tree-level parameter region where the Higgs mechanism occurs at very high energies and the adjoint matter field Φ is quite heavy. Then the massive particles are integrated out and the scale matching relation becomes

$$\Lambda_L{}^{6-N_f} = g_{N_c}^2 \Lambda^{2N_c - N_f}, \tag{2}$$

where Λ is the dynamical scale of high-energy $SU(N_c)$ theory with N_f flavors and Λ_L is the scale of low-energy $SU(2)$ theory with N_f flavors. Eq.(2) is derived by following the $SU(N_c)$ Yang-Mills case [8] while taking into account the existence of fundamental flavors at low energies [17].

The semiclassical superpotential in low-energy $SU(2)$ theory with N_f flavors reads

$$W = \sum_{n=1}^{N_c} g_n u_n^{cl} + \sum_{l=0}^{r} a_1^l \, \mathrm{Tr}_{N_f} \, \lambda_l \, \tilde{Q} Q \tag{3}$$

which is obtained by substituting the classical values of Φ and integrating out all the fields except for those coupled to the $SU(2)$ gauge boson. Here, the constraint $\mathrm{Tr}\Phi^{cl} = a_1 + \sum_{i=1}^{N_c-1} a_i = 0$ and the classical equation of motion $\sum_{i=1}^{N_c-1} a_i = -g_{N_c-1}/g_{N_c}$ yield [11]

$$a_1 = \frac{g_{N_c-1}}{g_{N_c}}. \tag{4}$$

Below the flavor masses which can be read off from the superpotential (3), the low-energy theory becomes $N=1$ $SU(2)$ Yang-Mills theory with the superpotential

$$W = \sum_{n=1}^{N_c} g_n u_n^{cl}. \tag{5}$$

This low-energy theory has the dynamical scale Λ_{YM} which is related to Λ through

$$\Lambda_{YM}{}^6 = \det \left(\sum_{l=0}^{r} \lambda_l a_1^l \right) g_{N_c}^2 \Lambda^{2N_c - N_f}. \tag{6}$$

As in the previous literatures [8],[9] we simply assume here that the superpotential (5) and the scale matching relation (6) are exact for any values of the tree-level parameters. Now we add to (5) a dynamically generated piece which arises from gaugino condensation in $SU(2)$ Yang-Mills theory. The resulting effective superpotential W_L where all the matter fields have been integrated out is thus given by

$$
\begin{aligned}
W_L &= \sum_{n=1}^{N_c} g_n u_n^{cl} \pm 2\Lambda_{YM}^3 \\
&= \sum_{n=1}^{N_c} g_n u_n^{cl} \pm 2 g_{N_c} \sqrt{A(a_1)}
\end{aligned} \tag{7}
$$

with A being defined as $A(x) \equiv \Lambda^{2N_c - N_f} \det \left(\sum_{l=0}^{r} \lambda_l x^l \right)$. From $\langle u_n \rangle = \partial W_L / \partial g_n$ we find

$$
\langle u_n \rangle = u_n^{cl}(g) \pm \delta_{n,N_c-1} \frac{A'(a_1)}{\sqrt{A(a_1)}} \pm \delta_{n,N_c} \frac{1}{\sqrt{A(a_1)}} \left(2A(a_1) - a_1 A'(a_1) \right). \tag{8}
$$

If we define a hyperelliptic curve

$$
y^2 = P(x)^2 - 4A(x), \tag{9}
$$

where $P(x) = \langle \det(x - \Phi) \rangle$ is the characteristic equation of Φ, the curve is quadratically degenerate at the vacuum expectation values (8). This can be seen by plugging (8) in $P(x)$

$$
P(x) = P_{cl}(x) \mp x \frac{A'(a_1)}{\sqrt{A(a_1)}} \mp \frac{1}{\sqrt{A(a_1)}} \left(2A(a_1) - a_1 A'(a_1) \right), \tag{10}
$$

where $P_{cl}(x) = \det(x - \Phi_{cl})$, and hence

$$
P(a_1) = \mp 2\sqrt{A(a_1)}, \qquad P'(a_1) = \mp \frac{A'(a_1)}{\sqrt{A(a_1)}}. \tag{11}
$$

Then the degeneracy of the curve is confirmed by checking $y^2|_{x=a_1} = 0$ and $\frac{\partial}{\partial x} y^2|_{x=a_1} = 0$.

The transition points from the confining to the Coulomb phase are reached by taking the limit $g_i \to 0$ while keeping the ratio g_i/g_j fixed [8]. These points correspond to the singularities in the moduli space. Therefore the curve (9) is regarded as the curve relevant to describe the Coulomb phase of the theory with the tree-level superpotential $W = \sum_{l=0}^{r} \mathrm{Tr}_{N_f} \lambda_l \tilde{Q} \Phi^l Q$. Indeed, the curve (9) agrees with the one obtained in [16].

Especially in the parameter region that has $N=2$ supersymmetry, we find agreement with the curves for $N=2$ $SU(N_c)$ QCD with $N_f<2N_c-1$ [12],[13],[15].[†]

The procedure discussed above can be also applied to the other classical gauge groups. Let us consider $N=1$ $SO(2N_c)$ supersymmetric gauge theory with an adjoint matter field Φ which is an antisymmetric $2N_c\times 2N_c$ tensor, and $2N_f$ flavors of fundamentals Q. We assume a tree-level superpotential

$$W=\sum_{n=1}^{N_c-2} g_{2n}u_{2n}+g_{2(N_c-1)}s_{N_c-1}+\lambda v+\frac{1}{2}\sum_{l=0}^{r}\mathrm{Tr}_{2N_f}\,\lambda_l\,Q\Phi^l Q, \tag{12}$$

where $r\le 2N_c-1$,

$$u_{2n}=\frac{1}{2n}\mathrm{Tr}\,\Phi^{2n}, \qquad v=\mathrm{Pf}\,\Phi=\frac{1}{2^{N_c}N_c!}\epsilon_{i_1i_2j_1j_2\cdots}\Phi^{i_1i_2}\Phi^{j_1j_2}\cdots \tag{13}$$

and

$$ks_k+\sum_{i=1}^{k} is_{k-i}u_{2i}=0, \qquad s_0=-1, \qquad k=1,2,\cdots. \tag{14}$$

Here, ${}^t\lambda_l=(-1)^l\lambda_l$ and the $N=2$ supersymmetry is present when we set $(\lambda_0)^i_j=m^i_j$ where $[m,m^\dagger]=0$, $(\lambda_1)^i_j=\mathrm{diag}(i\sigma_2,i\sigma_2,\cdots)$ with $\sigma_2=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$, $(\lambda_l)^i_j=0$ for $l>1$ and all $g_i=0$.

As in the case of $SU(N_c)$, we concentrate on the unbroken $SU(2)\times U(1)^{N_c-1}$ vacua with $\Phi=\mathrm{diag}(a_1\sigma_2,a_1\sigma_2,a_2\sigma_2,a_3\sigma_2,\cdots,a_{N_c-1}\sigma_2)$ and $Q=0$. By virtue of using s_{N_c} instead of u_{2N_c} in (12) the degenerate eigenvalue of Φ_{cl} is expressed by g_i

$$a_1^2=\frac{g_{2(N_c-2)}}{g_{2(N_c-1)}} \tag{15}$$

as found for the $SO(2N_c+1)$ case [9]. Note that the superpotential (12) has no classical vacua with unbroken $SO(4)\times U(1)^{N_c-1}$ when $g_{2(N_c-2)}\neq 0$. We also note that the fundamental representation of $SO(2N_c)$ is decomposed into two fundamental representations of $SU(2)$ under the above embedding. It is then observed that the scale matching relation between the high-energy $SO(2N_c)$ scale Λ and the scale Λ_L of low-energy $SU(2)$ theory with $2N_f$ fundamental flavors is given by

$$\Lambda_L^{6-2N_f}=g^2_{2(N_c-1)}\Lambda^{4(N_c-1)-2N_f}. \tag{16}$$

[†]For $N_f=2N_c-1$ an instanton may generate a mass term and shift the bare quark mass in $A(x)$. If we include this effect the curve (9) completely agrees with the result in [15].

The superpotential for low-energy $N=1$ $SU(2)$ QCD with $2N_f$ flavors can be obtained in a similar way to the $SU(N_c)$ case, but the duplication of the fundamental flavors are taken into consideration. After some manipulations it turns out that the superpotential for low-energy $N=1$ $SU(2)$ QCD with $2N_f$ flavors is written as

$$W=\sum_{n=1}^{N_c-2} g_{2n}u_{2n}^{cl}+g_{2(N_c-1)}s_{N_c-1}^{cl}+\lambda v^{cl}+\sum_{l=0}^{r} a_1^l \,\mathrm{Tr}_{2N_f}\,\lambda_l\,\widetilde{\mathbf{Q}}\mathbf{Q}, \tag{17}$$

where

$$\mathbf{Q}^j=\frac{1}{\sqrt{2}}\begin{pmatrix} Q_1^j-iQ_2^j \\ Q_3^j-iQ_4^j \end{pmatrix}, \qquad \widetilde{\mathbf{Q}}_j=\frac{1}{\sqrt{2}}\begin{pmatrix} Q_1^j+iQ_2^j \\ Q_3^j+iQ_4^j \end{pmatrix}. \tag{18}$$

Upon integrating out the $SU(2)$ flavors we have the scale matching between Λ and Λ_{YM} for $N=1$ $SU(2)$ Yang-Mills theory

$${\Lambda_{YM}}^6=\det\left(\sum_{l=0}^{r}\lambda_l a_1^l\right)\, g_{2(N_c-1)}^2\Lambda^{4(N_c-1)-2N_f}, \tag{19}$$

and we get the effective superpotential

$$\begin{aligned} W_L &= \sum_{n=1}^{N_c-2} g_n u_n^{cl}+g_{2(N_c-1)}s_{N_c-1}^{cl}+\lambda v^{cl}\pm 2\Lambda_{YM}^3 \\ &= \sum_{n=1}^{N_c-2} g_n u_n^{cl}+g_{2(N_c-1)}s_{N_c-1}^{cl}+\lambda v^{cl}\pm 2g_{2(N_c-1)}\sqrt{A(a_1)}, \end{aligned} \tag{20}$$

where A is defined by $A(x)\equiv\Lambda^{4(N_c-1)-2N_f}\det\left(\sum_{l=0}^{r}\lambda_l x^l\right)=A(-x)$.

The vacuum expectation values of gauge invariants are obtained from W_L as

$$\begin{aligned} \langle s_n\rangle &= s_n^{cl}(g)\pm\delta_{n,N_c-2}\frac{A'(a_1)}{\sqrt{A(a_1)}}\pm\delta_{n,N_c-1}\frac{1}{\sqrt{A(a_1)}}\left(2A(a_1)-a_1^2A'(a_1)\right), \\ \langle v\rangle &= v^{cl}(g), \end{aligned} \tag{21}$$

where $A'(x)=\frac{\partial}{\partial x^2}A(x)$. It is now easy to see that a curve

$$y^2=P(x)^2-4x^4A(x) \tag{22}$$

with $P(x)=\langle\det(x-\Phi)\rangle$ is degenerate at these values of $\langle s_n\rangle$, $\langle v\rangle$, and reproduces the known $N=2$ curve [14], [15].

The only difference between $SO(2N_c)$ and $SO(2N_c+1)$ is that the gauge invariant $\mathrm{Pf}\,\Phi$ vanishes for $SO(2N_c+1)$. Thus, taking a tree-level superpotential

$$W = \sum_{n=1}^{N_c-1} g_{2n} u_{2n} + g_{2N_c} s_{N_c} + \frac{1}{2} \sum_{l=0}^{r} \mathrm{Tr}_{2N_f}\, \lambda_l\, Q \Phi^l Q, \qquad r \leq 2N_c, \tag{23}$$

we focus on the unbroken $SU(2) \times U(1)^{N_c-1}$ vacuum which has the classical expectation values $\Phi = \mathrm{diag}(a_1\sigma_2, a_1\sigma_2, a_2\sigma_2, \cdots, a_{N_c-1}\sigma_2, 0)$ and $Q = 0$ [9]. As in the $SO(2N_c)$ case we make use of the scale matching relation between the high-energy scale Λ and the low-energy $N = 1$ $SU(2)$ Yang-Mills scale Λ_{YM}

$${\Lambda_{YM}}^6 = \det \left(\sum_{l=0}^{r} \lambda_l a_1^l \right) g_{2N_c} g_{2(N_c-1)} \Lambda^{2(2N_c-1-N_f)}. \tag{24}$$

As a result we find the effective superpotential

$$\begin{aligned} W_L &= \sum_{n=1}^{N_c-1} g_{2n} u_n^{cl} + g_{2N_c} s_{N_c}^{cl} \pm 2\Lambda_{YM}^3 \\ &= \sum_{n=1}^{N_c-1} g_{2n} u_n^{cl} + g_{2N_c} s_{N_c}^{cl} \pm 2\sqrt{g_{2N_c} g_{2(N_c-1)} A(a_1)}, \end{aligned} \tag{25}$$

where A is defined as $A(x) \equiv \Lambda^{2(2N_c-1-N_f)} \det \left(\sum_{l=0}^{r} \lambda_l x^l \right)$.

Noting the relation $a_1^2 = g_{2(N_c-1)}/g_{2N_c}$ [9] we calculate the vacuum expectation values of gauge invariants

$$\begin{aligned} \langle s_n \rangle = s_n^{cl}(g) \;&\pm\; \delta_{n,N_c-1} \frac{1}{\sqrt{A(a_1)}} \left(\frac{A(a_1)}{a_1} + a_1 A'(a_1) \right) \\ &\pm\; \delta_{n,N_c} \frac{1}{\sqrt{A(a_1)}} \left(a_1 A(a_1) - a_1^3 A'(a_1) \right). \end{aligned} \tag{26}$$

For these $\langle s_n \rangle$ we observe the quadratic degeneracy of the curve

$$y^2 = \left(\frac{1}{x} P(x) \right)^2 - 4x^2 A(x), \tag{27}$$

where $P(x) = \langle \det(x - \Phi) \rangle$. In the $N = 2$ limit we see agreement with the curve constructed in [14],[15]. The confining phase superpotential for the $SO(5)$ gauge group was obtained also in [10].

Let us now turn to $Sp(2N_c)$ gauge theory. We take for matter content an adjoint field Φ and $2N_f$ fundamental fields Q. The $2N_c \times 2N_c$ tensor Φ is subject to ${}^t\Phi = J\Phi J$ with $J = \mathrm{diag}(i\sigma_2, \cdots, i\sigma_2)$. Our tree-level superpotential reads

$$W = \sum_{n=1}^{N_c-1} g_{2n} u_{2n} + g_{2N_c} s_{N_c} + \frac{1}{2} \sum_{l=0}^{r} \mathrm{Tr}_{2N_f}\, \lambda_l\, QJ\Phi^l Q, \tag{28}$$

where ${}^t\lambda_l = (-1)^{l+1}\lambda_l$ and $r \leq 2N_c - 1$. The classical vacuum with the unbroken $SU(2) \times U(1)^{N_c-1}$ gauge group corresponds to

$$J\Phi = \mathrm{diag}(\sigma_1 a_1,\ \sigma_1 a_1,\ \sigma_1 a_2, \cdots, \sigma_1 a_{N_c-1}), \qquad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{29}$$

where $a_1^2 = g_{2(N_c-1)}/g_{2N_c}$. The scale Λ_L for low-energy $SU(2)$ theory with $2N_f$ flavors is expressed as [9]

$$\Lambda_L{}^{6-2N_f} = \left(\frac{g_{2N_c}^2}{g_{2(N_c-1)}} \right)^2 \Lambda^{2(2N_c+2-N_f)}. \tag{30}$$

There exists a subtle point in the analysis of $Sp(2N_c)$ theory. When $Sp(2N_c)$ is broken to $SU(2) \times U(1)^{N_c-1}$ the instantons in the broken part of the gauge group play a role since the index of the embedding of the unbroken $SU(2)$ in $Sp(2N_c)$ is larger than one (see eq.(30)) [18],[19]. The possible instanton contribution to W_L will be of the same order in Λ as low-energy $SU(2)$ gaugino condensation. Therefore even in the lowest quantum corrections the instanton term must be added to W_L.

For clarity we begin with discussing $Sp(4)$ Yang-Mills theory. In this theory by the symmetry and holomorphy the effective superpotential is determined to take the form $W_L = f\left(\frac{g_4}{g_2}\Lambda^2\right)\frac{g_4^2}{g_2}\Lambda^6$ with f being certain holomorphic function. If we assume that there is only one-instanton effect, the precise form of W_L including the low-energy gaugino condensation effect may be given by

$$W_L = 2\frac{g_4^2}{g_2}\Lambda^6 \pm 2\frac{g_4^2}{g_2}\Lambda^6, \tag{31}$$

as in the case of $SO(4) \simeq SU(2) \times SU(2)$ breaking to the diagonal $SU(2)$. This is due to the fact $Sp(4) \simeq SO(5)$ and the natural embedding of $SO(4)$ in $SO(5)$. Our low-energy $SU(2)$ gauge group is identified with the one diagonally embedded in $SO(4) \simeq SU(2) \times SU(2)$ [18],[20]. Accordingly, in $Sp(2N_c)$ Yang-Mills theory, we first make the

matching at the scale of $Sp(2N_c)/Sp(4)$ W bosons by taking all the $a_1 - a_i$ large. Then the low-energy superpotential is found to be

$$W_L = W_{cl} + 2\frac{g_{2N_c}}{a_1^2}\Lambda^{2(N_c+1)} \pm 2\frac{g_{2N_c}}{a_1^2}\Lambda^{2(N_c+1)}. \tag{32}$$

Let us turn on the coupling to fundamental flavors Q and evaluate the instanton contribution. When flavor masses vanish there is a global $O(2N_f) \simeq SO(2N_f) \times \mathbf{Z}_2$ symmetry. The couplings λ_l and instantons break a "parity" symmetry $\mathbf{Z}_2$. We treat this $\mathbf{Z}_2$ as unbroken by assigning odd parity to the instanton factor $\Lambda^{2N_c+2-N_f}$ and $O(2N_f)$ charges to λ_l. Symmetry consideration then leads to the one-instanton factor proportional to $B(a_1)$ where

$$B(x) = \Lambda^{2N_c+2-N_f}\,\mathrm{Pf}\left(\sum_{l\,\mathrm{even}} \lambda_l x^l\right). \tag{33}$$

Note that $B(x)$ is parity invariant since Pfaffian has odd parity. Thus the superpotential for low-energy $N=1$ $SU(2)$ QCD with $2N_f$ flavors including the instanton effect turns out to be

$$W = \sum_{n=1}^{N_c-1} g_{2n}u_{2n}^{cl} + g_{2N_c}s_{N_c}^{cl} + \sum_{l=0}^{r} a_1^l\,\mathrm{Tr}_{2N_f}\,\lambda_l\,\widetilde{\mathbf{Q}}\mathbf{Q} + 2\frac{g_{2N_c}^2}{g_{2(N_c-1)}}B(a_1), \tag{34}$$

where

$$\mathbf{Q}^j = \begin{pmatrix} Q_1^j \\ Q_3^j \end{pmatrix}, \qquad \widetilde{\mathbf{Q}}_j = \begin{pmatrix} Q_2^j \\ Q_4^j \end{pmatrix}. \tag{35}$$

When integrating out the $SU(2)$ flavors, the scale matching relation between Λ and the scale Λ_{YM} of $N=1$ $SU(2)$ Yang-Mills theory becomes

$${\Lambda_{YM}}^6 = \det\left(\sum_{l=0}^{r}\lambda_l a_1^l\right)\left(\frac{g_{2N_c}^2}{g_{2(N_c-1)}}\right)^2\Lambda^{2(2N_c+2-N_f)}, \tag{36}$$

and we finally obtain the effective superpotential

$$\begin{aligned} W_L &= \sum_{n=1}^{N_c-1} g_n u_n^{cl} + g_{2N_c}s_{N_c}^{cl} \pm 2\Lambda_{YM}^3 + 2\frac{g_{2N_c}^2}{g_{2(N_c-1)}}B(a_1) \\ &= \sum_{n=1}^{N_c-1} g_n u_n^{cl} + g_{2N_c}s_{N_c}^{cl} + 2\frac{g_{2N_c}^2}{g_{2(N_c-1)}}\left(B(a_1) \pm \sqrt{A(a_1)}\right), \end{aligned} \tag{37}$$

where $A(x) \equiv \Lambda^{2(2N_c+2-N_f)}\det\left(\sum_{l=0}^{r}\lambda_l x^l\right)$.

The gauge invariant expectation values $\langle s_n \rangle$ are

$$\begin{aligned}\langle s_n \rangle = s_n^{cl}(g) \quad &+ \quad \delta_{n,N_c-1}\frac{1}{a_1^4}\left(-2B(a_1) + 2a_1^2 B'(a_1) \pm \frac{1}{\sqrt{A(a_1)}}\Big(-2A(a_1) + a_1^2 A'(a_1)\Big)\right) \\ &+ \quad \delta_{n,N_c}\frac{1}{a_1^2}\left(4B(a_1) - 2a_1^2 B'(a_1) \pm \frac{1}{\sqrt{A(a_1)}}\Big(4A(a_1) - a_1^2 A'(a_1)\Big)\right). \end{aligned} \tag{38}$$

Substituting these into a curve

$$x^2 y^2 = \Big(x^2 P(x) + 2B(x)\Big)^2 - 4A(x), \tag{39}$$

we see that the curve is degenerate at (38). In this case too our result (39) agrees with the $N = 2$ curve obtained in [15].

Using the technique of confining phase superpotential we have determined the curves describing the Coulomb phase of $N = 1$ supersymmetric gauge theories with adjoint and fundamental matters with classical gauge groups. In the $N = 2$ limit our results recover the curves for the Coulomb phase in $N = 2$ QCD. For the gauge group $Sp(2N_c)$, in particular, we have observed that taking into account the instanton effect in addition to $SU(2)$ gaugino condensation is crucial to obtain the effective superpotential for the phase with a confined photon. This explains in terms of $N = 1$ theory a peculiar feature of the $N = 2$ $Sp(2N_c)$ curve when compared to the $SU(N_c)$ and $SO(N_c)$ cases.

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